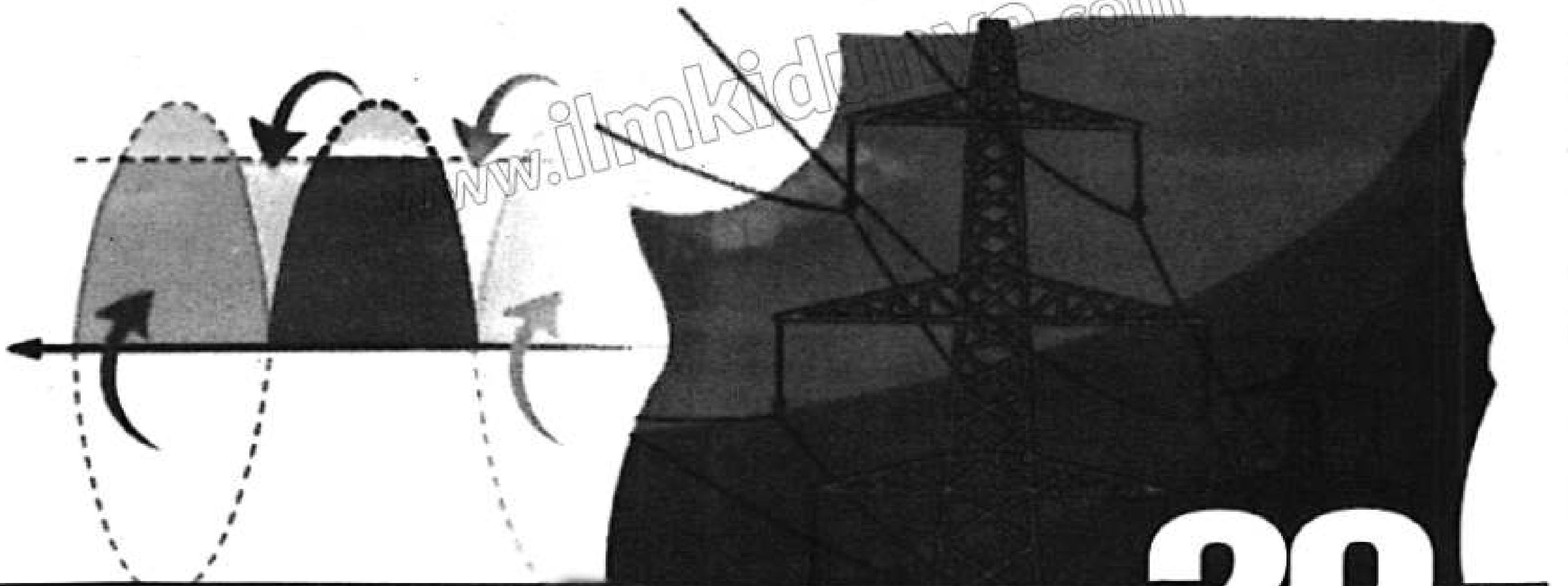


20 ALTERNATING CURRENT

Student Learning Outcomes (SLOs)

The student will

- use the terms period, frequency and peak value as applied to an alternating current or voltage.
- use equations of the form $x = x_o \sin(\omega t)$ representing a sinusoidally alternating current or voltage.
- use the fact that the mean power in a resistive load is half the maximum power for a sinusoidal alternating current.
- distinguish between root-mean-square (r.m.s.) and peak values [including stating and using $I_{rms} = I_o/\sqrt{2}$ and $V_{rms} = V_o/\sqrt{2}$ for a sinusoidal alternating current]
- Distinguish graphically between half-wave and full-wave rectification.
- explain the use of a single diode for the half-wave rectification of an alternating current.
- explain the use of four diodes (bridge rectifier) for full-wave rectification of an alternating current.
- analyze the effect of a single capacitor in smoothing current flow [including the effect of the values of capacitance and the load resistance].
- define mutual inductance (M) and self-inductance (L), and their unit henry.
- describe the phase of A.C and how phase lags and leads in A.C circuits.
- identify inductors as important components of A.C circuits termed as chokes [devices which present a high resistance to alternating current]
- Calculate the reactances of capacitors and inductors.
- describe impedance as vector summation of resistances and reactances.

The electric current used in electrical devices have two forms: Direct Current (DC) and Alternating Current (AC). Direct current flows continuously in one direction while alternating current changes its polarity at regular interval of time. The direction of alternating current at any instant depends upon the polarity of the voltages. When an alternating voltage is applied in a circuit, the current flows first in one direction and then in the opposite direction.

From the lights in your home to the smartphone in your pocket, alternating current (AC) plays a vital role in powering our daily lives. It is the backbone of modern electricity, powering our homes, industries, and technologies. But what makes AC so efficient and widely used? Have you ever wondered how does it transmit power efficiently over long distances? What makes it possible for us to use a wide range of electrical devices in our daily lives?

In this unit, we'll discuss about alternating current and its behaviour with resistor, capacitor and inductor. Through interactive examples, diagrams, and real-life applications, you'll gain a deeper understanding of the principles and technologies that underlie our modern electrical infrastructure. Get ready to unlock the secrets of alternating current and discover how it powers the modern world!

20.1 ALTERNATING CURRENT AND VOLTAGE

An ac generator gives alternating current or voltage of the form:

$$x = x_o \sin(\omega t)$$

Where, x represents magnitude of alternating current or voltage corresponding to the time t, and x_o represents maximum value of alternating current or voltage. ω represents the angular frequency of generator and hence of current and voltage.

For alternating current: $I = I_o \sin(\omega t)$ ______ (20.1 a)

For alternating voltage: $V = V_o \sin(\omega t)$ ______ (20.1 b)

The graph of voltage or current against time is shown in Fig. 20.1. This is a sine wave. A sinusoidal alternating voltage/current can be produced by rotating a coil with a constant angular velocity (ω) in a uniform magnetic field.

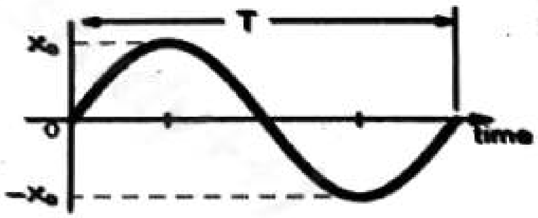


Figure 20.1: Sinusoidal wave form.

Graph shows that sinusoidal voltage or current:

- changes polarity (direction) after regular intervals.
- changes the magnitude continuously.
- the change from one polarity to the other is a smooth one.
- changing most rapidly at the zero (crossover) point and most slowly at its peak.

AC Terminologies

Some important A.C. terminology are defined below:

Cycle: One complete set of positive and negative values of an alternating quantity is known as a cycle. Figure 20.2 shows one cycle of an alternating voltage.

For Your Information
The frequency of A.C in Pakistan is 50 Hz.

Time Period (T): The time taken to complete one cycle of an alternating quantity is called its time period.

Frequency (f): The number of cycles that occurs in one second is called the frequency of the alternating quantity. It is measured in cycle/second or Hertz.

Average Value: The average value of a waveform is the average of all its values over a period of time. Finding an average value over time means adding all the values that occur in a specifying time interval and dividing the sum by that time.

The average value of a waveform from graph can be calculated by using the following formula:

$$\text{Average value} = \frac{\text{Total (net) area under the curve for time T}}{\text{Time T}}$$

The area above the time axis is taken as positive area and area below the time axis as negative area. In order to specify a sinusoidal voltage or current we do not use average value, because its value over one cycle is zero and cannot be used for power calculation.

Peak Value (x_o): Maximum value of alternating quantity (current or voltage) is called peak value, represents by x_o. The peak value of a sine wave occurs twice each cycle, once at the positive maximum value (x_o) and once at the negative maximum value ($-x_o$).

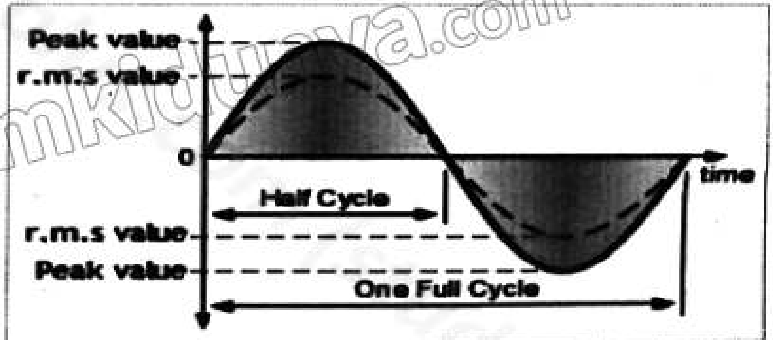


Figure 20.2: r.m.s and peak value.

Root-mean-square (r.m.s.) value: The r.m.s value of an alternating current is that steady current (d.c) which when flowing through a resistor produce the same amount of heat as that produced by the alternating current when flowing through the same resistance for the same time. The relation between root-mean-square (x_{rms}.) and peak values (x_o) is given below:

r.m.s value for alternating current: $x_{rms} = x_o/\sqrt{2}$

The relation for current and voltage is given as:

r.m.s value for alternating current: $I_{rms} = I_o/\sqrt{2}$ ______ (20.2 a)

r.m.s value for alternating voltage: $V_{rms} = V_o/\sqrt{2}$ ______ (20.2 b)

For example, if the effective or r.m.s value of an alternating current is 7 A, then the alternating current will produce the same heating effect as that produced by 7 A direct current.

Mean Power and Maximum Power

As the equation of the alternating current varying sinusoidally is given by:

$$I = I_o \sin\omega t$$

If this current is passed through a resistance R, then power delivered at any instant is

$$P = I^2 R$$

$$P = (I_o \sin\omega t)^2 R$$

or $$P = I_o^2 R \sin^2 \omega t$$

For Your Information

An alternating current or voltage can also be represented as a cosine function of time, i.e.,

$$I = I_o \cos\omega t$$

$$V = V_o \cos\omega t$$

As the current is squared, so power is always positive. Since the value of $\sin^2\omega t$ varies between 0 and 1, its average value is 1/2. So, the average power delivered can be expressed as:

$$< P > = \frac{1}{2} I_o^2 R \quad \text{\_\_\_\_\_\_ (20.3)}$$

This shows that the mean power in a resistive load is half the maximum power for a sinusoidal alternating current.

Example 20.1: An A.C. circuit consists of a pure resistance of 20 Ω and is connected across A.C. supply of 220 V, 50 Hz. Calculate (a) peak value of voltage (b) peak value of current (c) equation for voltage and current.

Given: R = 20 Ω $\quad V_{rms} = 220$ V $\quad$ f = 50Hz

To Find: a) $V_o = ?$ $\quad$ a) $I_o = ?$ $\quad$ b) Equation for V and I = ?

Solution: a) For peak value of an alternating voltage, we use:

$$V_{rms} = V_o/\sqrt{2} \quad \text{OR} \quad V_o = \sqrt{2}\, V_{rms}$$

Putting values, we get:

$$V_o = \sqrt{2} \times 220 = 311.1 \text{ V}$$

b) For peak value of an alternating voltage, we use:

$$I_o = V_o/R$$

Putting values, we get:

$$I_o = 311.1/20 = 15.55 \text{ A}$$

c) As, $\omega = 2\pi f = 2\pi \times 50 = 314$ rad/s

So, equation for voltage is:

$$V = V_o \sin\omega t$$

$$V = 311.1 \sin(314t)$$

Equation for current is:

$$I = I_o \sin\omega t$$

$$I = 15.55 \sin(314t)$$

Assignment 20.1

The peak voltage of an ac supply is 320 V. What is the rms value of this voltage?

20.2 RECTIFICATION

Every electronic circuit needs a dc voltage for its functioning. This dc voltage has been obtained from the ac supply. For this purpose, the ac supply voltage has to be reduced (stepped down) first using a step-down transformer and then converted to dc by using a circuit called rectifier.

The process of converting ac voltage into dc voltage is called rectification.

A rectifier circuit uses diode for rectification. There are two types of rectification processes; half wave rectification and full wave rectification.

20.2.1 Half Wave Rectification

In a half-wave rectification process an AC signal is converted into DC by passing one half-cycle of the waveform and blocking the second-half. Half-wave rectifiers can be easily constructed using one diode. A diode D is connected in series with the load resistance R, as shown in Fig. 20.3.

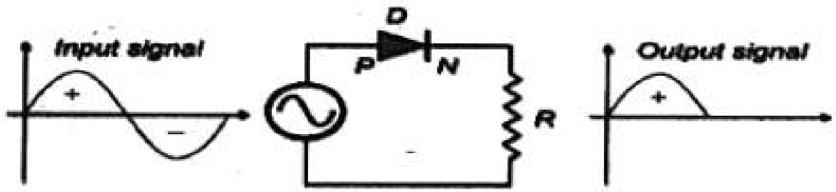


Figure 20.3: Half-wave rectifier circuit.

For the positive half cycle of the AC voltage, the diode D is forward biased, so it offers very low resistance and current flows through the resister R. Hence when the diode is forward biased, it acts as a closed switch, as shown below in the Fig. 20.4 (a).

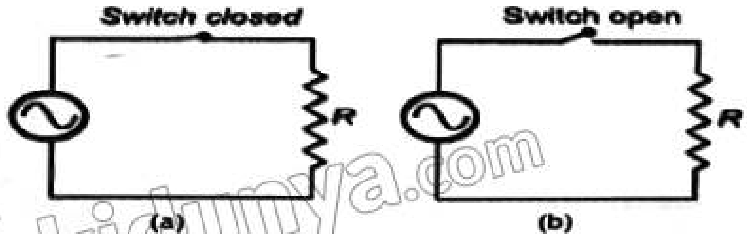


Figure 20.4: Effective circuit of half wave rectifier during: (a) forward biased (b) reversed biased.

For the negative half cycle of the AC voltage, the diode D is reversed biased, so it offers very high resistance and no current flows through the resister R (the output voltage is equal to zero). Hence when the diode is reversed biased, it acts as an open switch, as shown in the Fig. 20.4 (b). The half-wave rectifier's waveform before and after rectification is shown in the Fig. 20.5. The output waveform of a halfwave rectifier is a pulsating DC waveform. Filters in halfwave rectifiers are used to transform the pulsating DC waveform into constant DC waveforms. A capacitor can be used as a filter.

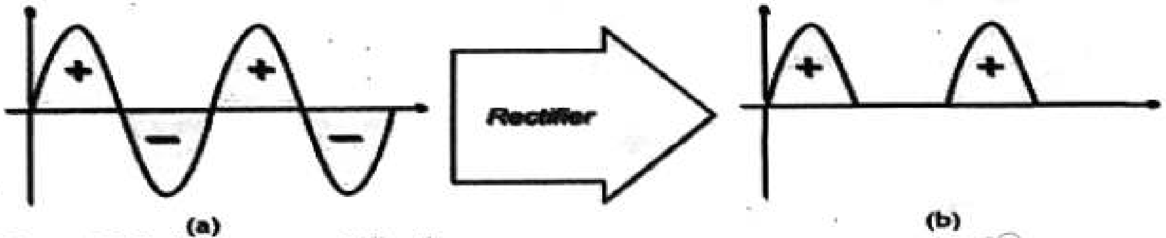


Figure 20.5: Half-wave rectification:
(a) input signal's waveform (b) output signal's waveform.

20.2.2 Full Wave Rectification

In full wave rectification, the complete cycle of AC signal is converted into pulsating DC. The circuit of the full wave rectifier can be constructed in two ways. The one method uses a centre-tapped transformer and two diodes. This arrangement is known as a centre-tapped full wave rectifier. Another method uses four diodes arranged as a bridge. This is known as a bridge rectifier. Here we will discuss only full wave bridge rectifier circuit.

Working of a Full Wave Bridge Rectifier: A full wave bridge rectifier is shown in the Fig. 20.6 (a). The circuit consists of four diodes D_1, D_2, D_3 and D_4 connected to form a bridge.

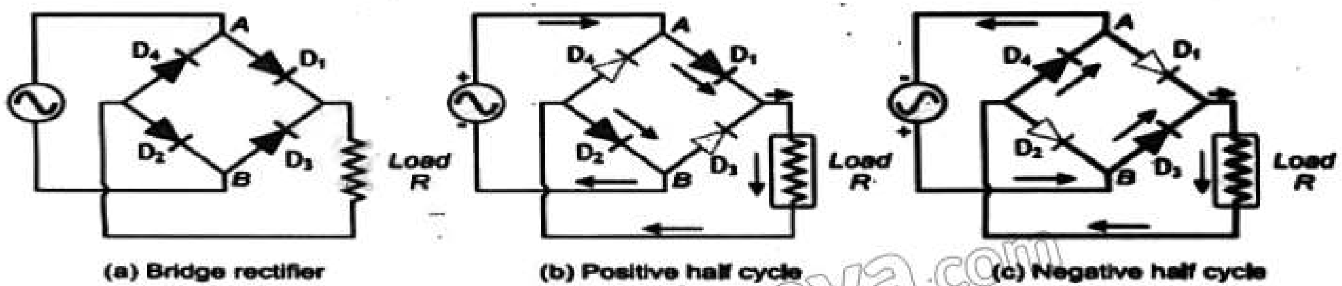


Figure 20.6: (a) Full wave bridge rectifier circuit. (b) Working of a full wave rectifier circuit during positive half cycle. (c) Working of a full wave rectifier circuit during negative half cycle. The arrows are showing the direction of the current flow.

During positive half cycle of secondary voltage, the end A becomes positive and end B is negative, as shown in Fig. 20.6 (b). This makes diodes D_1 and D_2 forward biased and diodes D_3 and D_4 reverse biased. Therefore, diodes D_1 and D_2 conduct while diodes D_3 and D_4 do not conduct. Thus, current (I) flows through diode D_1, load resistor R (from top to bottom), diode D_2 and to the negative terminal of input.

During negative half cycle, the end A becomes negative with respect to end B, as shown in Fig. 20.6 (c). This brings diodes D_3 and D_4 under forward bias and diodes D_1 and D_2 under reverse bias. Therefore, diodes D_3 and D_4 conduct while diodes D_1 and D_2 do not. Thus, current flows through diode D_3, load resistor R (from Top to bottom), diode D_4 and to the negative terminal of input, as shown in Fig. 20.6 (c).

It is obvious that one pair (D_1 and D_2) allows current flow during the positive half cycle of input voltage while the other pair (D_3 and D_4) allows current flow during the negative half cycle of input voltage. The current flowing through the load resistor R is in the same direction (top to bottom) during both half cycles. Hence, rectified output voltage is obtained across the load resistor R. The wave shape of input and output voltage is shown in Fig. 20.7.

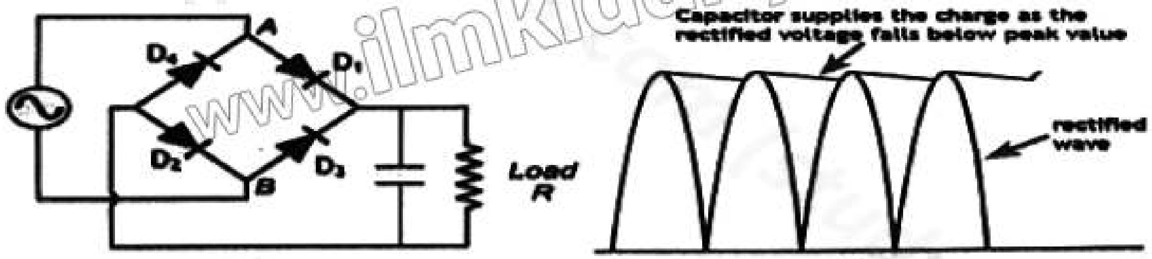
(a) (b)

Figure 20.7: Full-wave rectification:
(a) input signal's waveform (b) output signal's waveform.

Filtering

In a rectifier circuit (as shown in the Fig. 20.8-a), a capacitor smooths out the pulsating direct current (DC) into a more stable, constant output. This process is often referred to as 'filtering'.

As the voltage rises up to the peak of the input waveform, the capacitor charges to the peak voltage. When the input voltage drops, the capacitor discharges slowly by supplying charge to the load. The capacitor's discharge helps to fill in the gaps between the peaks of the rectified waveform, thereby reducing the voltage fluctuations and making the output smoother, as shown in the Fig. 20.8 (b).



(a) (b)

Figure 20.8: (a) Full-wave rectifier with capacitor: (b) smoothing action by capacitor.

The choice of capacitance value and load resistance depends on the specific application requirements. A higher capacitance value provides better smoothing, but may increase the charging current. A lower load resistance causes the capacitor to discharge faster, reducing its ability to smooth the output.

20.3 MUTUAL INDUCTANCE AND SELF-INDUCTANCE

Self-induction: Consider a coil, which is connected to a source of emf say a battery, through a switch and a rheostat, as shown in Fig. 20.9 (a). If we move the rheostat quickly, the current through the coil will change with time. This change in current in the coil, changes the magnetic field, hence the flux through the coil changes, which finally induces an emf in the coil itself.

The phenomenon, in which a changing current induces an emf in the coil itself, is called self-induction.

Self-induced emf in a coil is proportional to time rate of change of current through the coil, i.e.,

$$\varepsilon_L = -L\frac{\Delta I}{\Delta t} \quad \text{_______ (i)}$$

Self-inductance of the coil 'L' can be defined as:

Self-inductance of a coil is the ratio of emf induced in the coil to the time rate of change of current through it.

Eq. (i) can also be written as:

$$L = \frac{-\varepsilon_L}{\Delta I/\Delta t} \quad \text{_______ (20.4)}$$

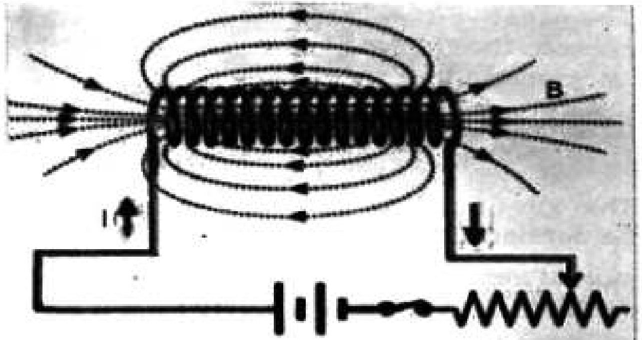


Figure 20.9 (a): Self-induction

The negative sign in eq. (20.4) is with accord of Lenz's law. It indicates that self-induced emf opposes the change, which produces it. due to this reason, self-induced emf is sometimes called as 'Back emf'. Due to self-inductance of coils, these are also known as 'Inductors' or 'choke'. Inductors are widely used in electrical technologies. In A.C. circuits they behave like resistors.

From the eq. (20.4), the unit of self-inductance is henry (H),

$$H = V\,s\,A^{-1}$$

In case of self-inductance, one henry can be defined as:

Self-inductance of a coil will be one henry, if one volt emf is induced in the coil by the change of current at the rate of one ampere per second.

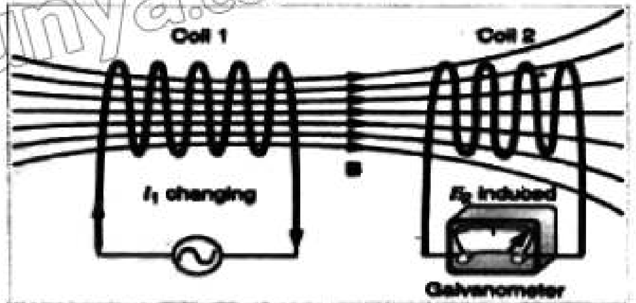


Figure 20.9 (b): Mutual induction between two coils

Mutual Induction: Consider two neighboring coils, as shown in Fig. 20.9 (b). One coil is connected to an emf source (A.C supply or D.C source with varying magnitude), while the other coil is connected to a galvanometer. The coils are placed near each other, such that the flux of coil-1 links with the coil-2. When A.C flows through coil-1 due to change of magnitude and direction of A.C, flux through the coil-1 changes, as this flux also links the coil-2, the changing flux through the coil-2 induces an emf in it, which can be seen by deflection of galvanometer's needle connected to the coil-2. This is called mutual induction, which can be defined as:

The phenomenon, in which changing current in one coil, induces an emf in neighboring coil, is called mutual induction.

The emf induced in coil-2 is proportional to time rate of change of current through the coil-1, i.e.,

$$\varepsilon_2 = -M\frac{\Delta I_1}{\Delta t} \quad \text{——— (ii)}$$

The negative sign in the equation shows, that the direction of induced current is such that, it opposes the change of current in the coil-1. From above relation the value of mutual inductance 'M' can also be written as:

$$M = \frac{\varepsilon_2}{\Delta I_1/\Delta t} \quad \text{——— (20.5)}$$

The unit of mutual-inductance is also henry (H). In case of mutual inductance, one henry can be defined as:

Mutual inductance between two coils is 1 henry if current changing at the rate of 1 A/s in one coil induces an emf of 1 V in the other coil.

20.4 PHASE OF A.C

In an AC circuit, the alternating current and voltage do not peak at the same time through capacitors or Inductors. Sometime voltage may be passing through its zero point while the current has passed or it is yet to pass through its zero point in the same direction. The angle between their zero points is the phase difference. The quantity which passes through its zero point earlier is said to leading while the other is said to be lagging. Since both alternating quantities have the same frequency, the phase difference between them remains the same.

The fraction of a period difference between the peaks expressed in degrees is said to be the phase difference.

It is represented by Φ. It is generally measured in degrees or radians. The phase difference is always less than or equal to 90° (It is customary to use the angle by which the voltage leads the current).

20.4.1 A.C. Through Resistor

Consider a circuit containing a resistor connected across an alternating voltage source as shown in Fig 20.10 (a).

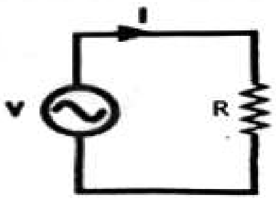


Figure 20.10 (a): A.C through a resistor.

The alternating voltage is be given by:

$$V = V_o \sin\omega t \quad \text{——— (i)}$$

Where, V_o is the peak value of the alternating voltage. As a result of this voltage, an alternating current 'I' will flow in the circuit. According to Ohm's law:

$$V = IR \quad \text{or} \quad I = \frac{V}{R}$$

or $$I = \frac{V_m \sin \omega t}{R}$$

As, $\frac{V_m}{R} = I_o$, so

$$I = I_o \text{ Sin}\omega t \quad \text{\_\_\_\_\_\_\_ (ii)}$$

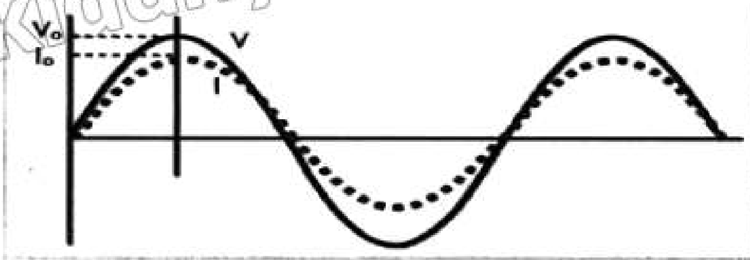


Figure 20.10 (b): Voltage and current are in phase.

Eqs. (i) and (ii) shows that:

In a resistor, applied voltage and the circuit current are in phase with each other.

Figure 20.10 (b) shows the waveform of current and voltage through a resistor. The applied voltage and current pass through their zero values at the same instant. The applied voltage and current attain their positive and negative peaks at the same instant. Hence current is in phase with the applied voltage. This is also indicated by the phasor diagram, as shown in Fig. 20.10 (c).

Figure 20.10 (c): Phasor diagram for resistor.

20.4.2 A.C Through an Inductor

An inductor is an electrical component which opposes changes in electric current passing through it. It consists of a conductor (such as a wire) usually wound into a coil.

Consider an alternating voltage is applied across an inductor of inductance L, as shown in Fig. 20.11 (a). Let the equation for alternating current is:

$$I = I_o \sin\omega t \quad \text{\_\_\_\_\_\_\_ (i)}$$

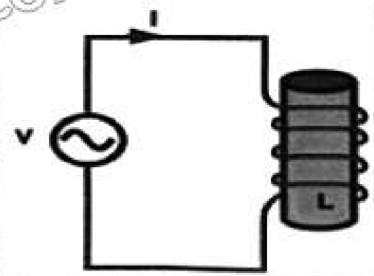


Figure 20.11 (a): A.C through an Inductor.

When a sinusoidal current 'I' flows in time t then a back e.m.f. ($=L\Delta I/\Delta t$) is induced due to the inductance of the coil. This back e.m.f. at every instant opposes the change in current through the coil. As there is no drop in potential across the inductor (because resistance of an ideal inductor is zero), so the applied voltage has to overcome the back emf, i.e.,

$$\text{Applied alternating voltage} = \text{Back emf} \quad \text{\_\_\_\_\_\_\_ (ii)}$$

So, the energy which is required in building up current in inductance L, is returned back during the decay of the current. The changing current sets up a back e.m.f in the coil. The magnitude of back e.m.f is:

$$\varepsilon = L\,\frac{\Delta I}{\Delta t}$$

Using Eq. (i) and Eq. (ii), the magnitude of applied voltage is:

For Your Information

A choke is an inductor used in a circuit. It offers high reactance to frequencies above a certain frequency range, without appreciable limiting the flow of current.

$$V = L\frac{\Delta I}{\Delta t} = L\frac{\Delta(I_o \sin\omega t)}{\Delta t}$$

As, $\frac{\Delta(\sin\omega t)}{\Delta t} = \omega \cos\omega t$,

So, $$V = L I_o \frac{\Delta(\sin\omega t)}{\Delta t}$$

$$V = \omega L I_o \cos\omega t$$

or $$V = V_o \cos\omega t$$

or $$V = V_o \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{\_\_\_\_\_\_ (20.6)}$$

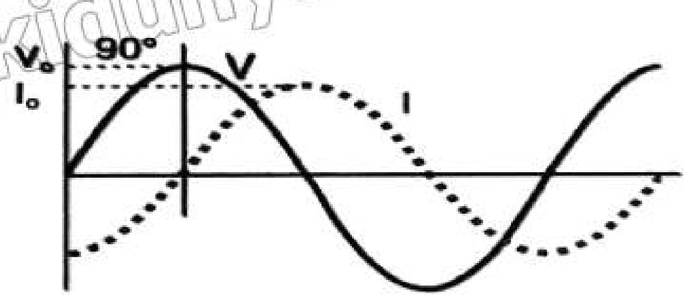


Figure 20.11(b): Voltage leads the current by 90°.

From Eq. (i) and Eq. (20.6), it is clear that:

In an inductor, voltage leads the current by π/2 radians or 90°.

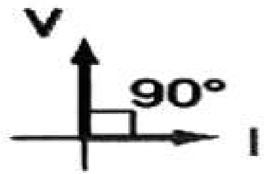


Figure 20.11 (c): Phasor diagram for inductor.

Figure 20.11 (b) also shows that current lags the voltage in an inductive coil. Inductance opposes the change in current and serves as increase or decrease of current in the circuit. This causes the current to lag behind the applied voltage which is indicated by the phasor diagram shown in Fig. 20.11 (c).

Inductive Reactance (X_L): The opposition offered by an inductor to the flow of A.C. is called inductive reactance. Therefore, in analogy to Ohm's law we can write:

$$V_o = I_o X_L$$

or $$X_L = \frac{V_o}{I_o}$$

or $$X_L = \frac{I_o \omega L}{I_o} \qquad (\text{using } V_o = I_o \omega L)$$

$$X_L = \omega L \qquad \text{or} \qquad X_L = 2\pi f L \quad \text{\_\_\_\_\_\_ (20.7)}$$

Hence, reactance of a coil depends upon frequency of A.C. In case of D.C, $f = 0$ so $X_L = 0$. Inductance has the same dimensions as resistance; therefore, it is measured in Ω.

20.4.2 A.C Through a Capacitor

Consider an alternating voltage is applied across a capacitor of capacitance C, as shown in Fig. 20.12 (a). Due to the alternating voltage, the capacitor is charged in one direction and then in the other as the voltage reverses. The result is that electron move to-and-fro around the circuit, connecting the plates, thus constituting alternating current. The basic

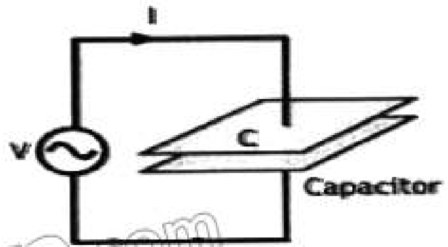


Figure 20.12 (a): A.C through a capacitor.

relation between the charge q on the capacitor and voltage V across its plates i.e. $q = CV$ holds at every instant. Let the equation for the applied alternating voltage is:

$$V = V_o \sin\omega t \quad \text{\_\_\_\_\_\_ (i)}$$

Then, at any instant charge on capacitor is:

$$q = CV = C\,V_o \sin\omega t$$

Now current 'I' flowing through capacitor is:

$$I = \frac{\Delta q}{\Delta t} = \frac{\Delta(CV_o \sin\omega t)}{\Delta t}$$

$$I = CV_o\,\omega \cos(\omega t)$$

$$I = CV_o\,\omega \sin\left(\omega t + \frac{\pi}{2}\right)$$

Here $I_o = CV_o\,\omega$, so

$$I = I_o \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{\_\_\_\_\_\_ (20.8)}$$

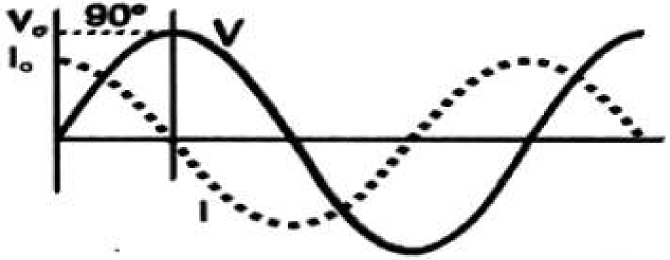


Figure 20.12 (b): Voltage lags the current by 90°.

From the Eq. (i) and Eq. (20.8), it is clear that:

In a capacitor, voltage lags the current by π/2 radians or 90°.

Capacitance opposes the change in voltage and serves to delay the increase or decrease of voltage across capacitor.

The phasor diagram in Fig. 20.12 (c) shows that in a capacitor the voltage lags behind the current.

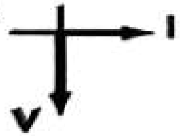


Figure 20.12 (c): Phasor diagram for capacitor.

Capacitive Reactance (X_C): The opposition offered by a capacitor to the flow of A.C is called capacitive reactance. Therefore, in analogy to Ohm's law, we can write:

$$V_o = I_o X_C$$

or $\quad X_C = \frac{V_o}{I_o} \quad$ or $\quad X_C = \frac{V_o}{CV_o\omega} \quad$ (As for capacitor, $I_o = C\,V_o\,\omega$)

$$X_C = \frac{1}{C\omega} \quad \text{or} \quad X_C = \frac{1}{2\pi fC} \quad \text{\_\_\_\_\_\_ (20.9)}$$

The capacitive reactance depends upon frequency of A.C. In case of D.C, X_C has infinite value. Capacitive reactance has the same dimensions as resistance; therefore, it is measured in Ω.

Mnemonic 'ELI the ICE man'

The mnemonic 'ELI the ICE man' helps to remember the phase difference of voltage and current in capacitor and inductor.

20.5 IMPEDANCE

Impedance is the combine effect of the resistance and the reactance present in an AC circuit. Thus, impedance can be broken down into resistance and reactance. Impedance is equivalent to resistance in AC circuit. These two provide opposition to the flow of alternating current in the circuit, and are generated due to the presence of capacitor and inductor. Let us explore impedance in RL and RC series circuits:

RL Series AC Circuit

Consider an AC circuit in which a resistor (R) is connected in series with a coil of pure inductance (L), as shown in Fig. 20.13 (a). Here, the voltage 'V' will be the phasor sum of the two component voltages, V_R and V_L. This means that the current flowing through the coil will still lag the voltage, but by an amount less than 90° depending upon the values of V_R and V_L. Taking current as the reference phasor, the phasor diagram of the circuit can be drawn, as shown in Fig. 20.13 (b).

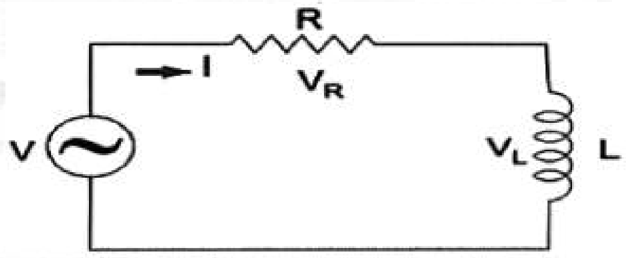


Figure 20.13 (a): RL series circuit.

- The voltage drops, $V_R = IR$ is in phase with current and is represented by the phasor OA.
- The voltage drops $V_L = IX_L$ leads the current by 90° and is represented the phasor AB.

The applied voltage V is the phasor sum of these two drops i.e.,

$$V^2 = V_R^2 + V_L^2$$

or

$$V = \sqrt{V_R^2 + V_L^2}$$

$$V = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I\sqrt{R^2 + X_L^2}$$

$$\frac{V}{I} = \sqrt{R^2 + X_L^2}$$

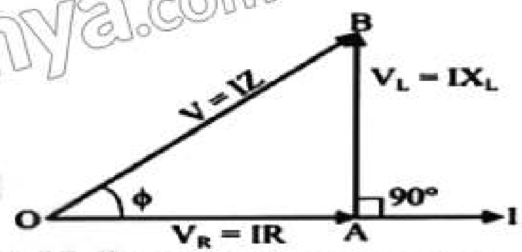


Figure 20.13 (b): Phasor diagram of RL series circuit.

As, V/I represent impedance and is denoted by Z. So,

$$Z = \sqrt{R^2 + X_L^2} \qquad (20.10)$$

Where $X_L = 2\pi f L$. RL Series Impedance triangle is shown in Fig. 12.13 (c), having sides *R, X_L and Z*. The magnitude of impedance in R-L series circuit depends upon the values of resistance R, inductance L and supply frequency f.

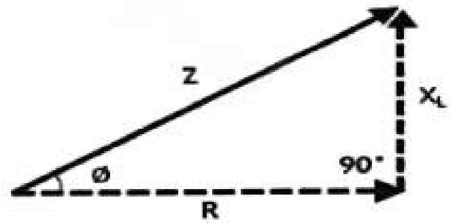


Figure 20.13 (c): RL Series Impedance triangle.

RC Series AC Circuit

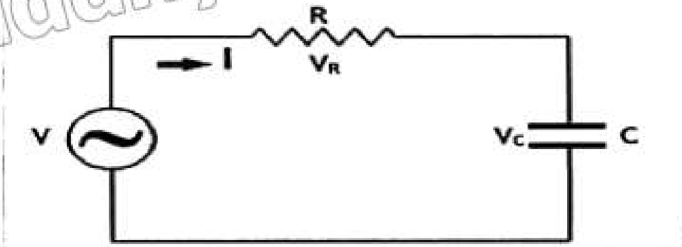


Figure 20.14 (a): RC series circuit.

Consider an AC circuit in which a capacitor (C) and resistor (R) are connected in series with each other, as shown in Fig 20.14 (a).

The voltage 'V' across the combination is equal to the phasor sum of two component voltages, $V_R = IR$ & $V_C = IX_C$. Taking current as the reference phasor, the phasor diagram of the circuit can be drawn, as shown in Fig 20.14 (b).

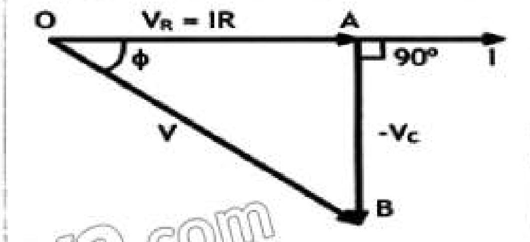


Figure 20.14 (b): Phasor diagram of RC series circuit.

The voltage drop V_R (= IR) is in phase with current and is represented by the phasor OA.

The voltage drop V_C (= IX_C) lags behind the current by 90° and is represented by the phasor AB. The applied voltage V is the phasor sum of these two potential drops i.e.,

$$V^2 = V_R^2 + V_C^2$$

or

$$V = \sqrt{V_R^2 + V_C^2}$$

$$V = \sqrt{(IR)^2 + (-IX_C)^2}$$

$$V = I\sqrt{R^2 + X_C^2}$$

$$\frac{V}{I} = \sqrt{R^2 + X_C^2}$$

As, V/I represent impedance and is denoted by Z. So,

$$Z = \sqrt{R^2 + X_C^2} \quad \text{\_\_\_\_\_\_ (20.11)}$$

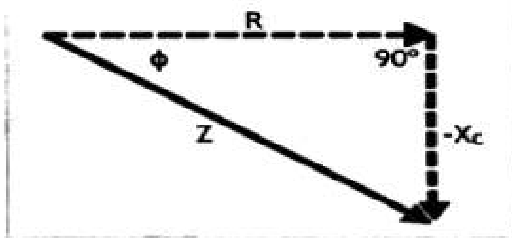


Figure 20.13 (c): RC series impedance triangle.

Where $X_C = \frac{1}{C\omega}$. Impedance Z is measured in ohms (Ω).

Example 20.2: A.C voltage across a 0.5 μF capacitor is $16\sin(2\times10^3 t)$ V. Find (a) the capacitive reactance (b) the peak value of current through the capacitor.

Given: $C = 0.5\ \mu F = 0.5\times10^{-6}$ F $\qquad V = 16\sin(2\times10^3 t)$ V

To Find: a) X_C = ? $\qquad$ b) I_o = ?

Solution: a) As alternating voltage is given by relation

$$V = V_o \sin(\omega t) \quad \text{\_\_\_\_\_\_ (i)}$$

Also given that

$$V = 16\sin(2\times10^3 t) \quad \text{\_\_\_\_\_\_ (ii)}$$

Comparing (i) and (ii), we get:

$$V_o = 16\ V \quad \text{and} \quad \omega = 2\times10^3$$

So, $X_C = \dfrac{1}{C\omega} = \dfrac{1}{(0.5\times10^{-6})(2\times10^{3})} = 1000\ \Omega$

b) Using ohm's law:

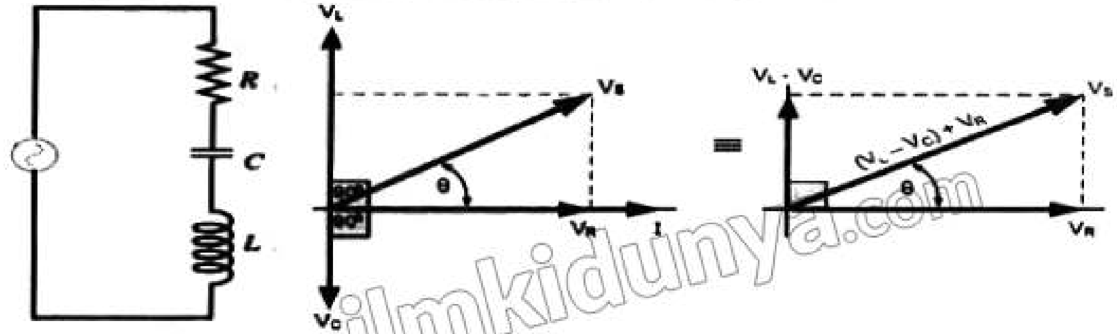
$$I_o = \frac{V_o}{X_c} = \frac{16}{1000} = 16\times10^{-3}\ \text{A} = 16\ \text{mA}$$

Assignment 20.2

The voltage across a 0.01 μF capacitor is 240 sin($1.25\times10^4 t$ − 30°) V. Write the mathematical expression for the current through it.

For Your Information

A circuit in which a resistor (R), an inductor (L), and a capacitor (C) are connected in series is called RLC series circuit. All the elements of RLC series circuit share the same current. The RLC series circuit and its Phasor diagram is given below:



SUMMARY

- One complete set of positive and negative values of an alternating quantity is known as a cycle.
- The time taken to complete one cycle of an alternating quantity is called its time period.
- The number of cycles that occurs in one second is called the frequency of the alternating quantity. It is measured in cycle/second or Hertz.
- The average value of a waveform is the average of all its values over a period of time.
- Maximum value of alternating quantity (current or voltage) is called peak value.
- The r.m.s value of an alternating current is that steady current (d.c.) which when flowing through a resistor produce the same amount of heat as that produced by the alternating current when flowing through the same resistance for the same time.
- The process of converting ac voltage into dc voltage is called rectification.
- In a half-wave rectification process an AC signal is converted into DC by passing one half-cycle of the waveform and blocking the second-half.
- In full wave rectification, the complete cycle of AC signal is converted into pulsating DC.

- In a rectifier circuit, a capacitor smooths out the pulsating direct current (DC) into a more stable, constant output. This process is often referred to as 'filtering'.
- The phenomenon, in which a changing current induces an emf in the coil itself, is called self-inductance. Self-inductance of a coil is the ratio of emf induced in the coil to the time rate of change of current through it.
- Self-inductance of a coil will be one henry, if 1 volt emf is induced in the coil by the change of current at the rate of one ampere per second.
- The phenomenon, in which changing current in one coil, induces emf in neighboring coil, is called mutual induction.
- Mutual inductance between two coils is 1 henry if current changing at the rate of 1 A/s in one coil induces an emf of 1 V in the other coil.
- The fraction of a period difference between the peaks expressed in degrees is said to be the phase difference.
- In an inductor, volage leads the current by $\frac{\pi}{2}$ radians or 90°.
- In a capacitor, volage lags the current by $\frac{\pi}{2}$ radians or 90°.
- A Choke is an inductor used in a circuit. It offers high reactance to frequencies above a certain frequency range, without appreciable limiting the flow of current.
- Impedance is the combine effect of the resistance and the reactance present in an AC circuit.

EXERCISE

Multiple Choice Questions

Encircle the Correct option.

1) A full wave bridge rectifier consists of
   A. no diode B. one diode C. Two diodes D. four diodes
2) Unit of reactance is
   A. henry B. ohms C. volt D. ampere
3) The unit of self-inductance, henry (H), is equal to:
   A. $A\,s^{-1}\,V^{-1}$ B. $V\,A\,s^{-1}$ C. $V\,s^{-1}\,A^{-1}$ D. $V\,s\,A^{-1}$
4) A capacitor is perfectly insulator for
   A. direct current
   B. alternating current
   C. direct as well as alternating current
   D. neither for direct current nor for alternating current
5) The peak value of alternating current is $5\sqrt{2}$ A. The mean square value of current will be
   A. 5 A B. 2.5 A C. $5\sqrt{2}$ A D. 5^2 A
6) In AC system we generate sine wave form because
   A. It can be easily draw B. It produces least disturbance in electrical circuits

C. It is nature standard D. Other waves cannot be produced easily.

7) An alternating voltage is given by 20sin(157t). The frequency of alternating voltage is
A. 50 Hz B. 25 Hz C. 100 Hz D. 75 Hz

8) Peak value of an alternating quantity (voltage or current) can be found if we know it's
A. phase B. r.m.s value C. frequency D. both (a) & (b)

Short Questions

1) Define (a) mutual-induction (b) self-induction.
2) Does the SI unit used for mutual-inductance and self-inductance are same? Explain briefly.
3) Draw the circuit for a half wave rectifier and full wave rectifier.
4) What is the use of a single diode for the half-wave rectification of an alternating current?
5) Distinguish graphically between half-wave and full-wave rectification.
6) Is the frequency content of the output of a half wave rectifier and full wave rectifier the same? Explain briefly.
7) In half wave rectifier, the half of the signal is blocked after the rectification. Why?
8) Distinguish between root-mean-square (r.m.s.) and peak values for a sinusoidal alternating current.
9) What is impedance? How is it related to resistance, reactance, and frequency? Also find its SI unit?
10) What is the difference between reactance and impedance?
11) Why does a capacitor block DC but allow AC to pass through?
12) How does the reactance of a capacitor change with an increase in frequency?
13) What is the phase difference between voltage and current in a (a) capacitor (b) Inductor?
14) How does the capacitance of a capacitor affect the current through it in an AC circuit?

Comprehensive Questions

1) What is half wave rectification? Explain.
2) With the help of a diagram, explain the operation of a bridge rectifier.
3) Analyze the effect of a single capacitor in smoothing current flow.
4) Explain the phenomenon of self-induction.
5) A sinusoidal alternating voltage of angular frequency ω is connected across an inductor. Find mathematical expression for instantaneous voltage and instantaneous current.
6) Explain the term impedance of an AC circuit. Find its expression for the RL series circuit.
7) In an RL series circuit will the current lag or lead the applied alternating voltage? Explain the answer with a phasor diagram.

8) Describe the phase of A.C and how phase lags and leads in A.C circuits.

9) Describe impedance as vector summation of resistances and reactances.

Numerical Problems

1) What is the rms value of the voltage of an ac supply having peak voltage 300 V? (Ans: 212 V)

2) An inductor with an inductance of 100 μH passes a current of 10 mA when its terminal voltage is 6.3 V. Calculate the frequency of A.C supply. (Ans: 10^6 Hz)

3) A coil of pure inductance 318 mH is connected in series with a pure resistance of 75 Ω. The voltage across resistor is 150 V and the frequency of power supply is 50 Hz. Calculate impedance of the circuit. (Ans: 124.9 Ω)

4) A resistor of resistance 30 Ω is connected in series with a capacitor of capacitance 79.5 μF across a power supply of 50 Hz and 100 V. Find impedance of the circuit. (Ans: 50 Ω)

Student Learning Outcomes (SLOs)

The student will

- state that electromagnetic radiation has a particulate nature.
- Explain and apply the photonic model of light to solve problems [use E = hf to solve problems, and use the electronvolt (eV) as a unit of energy]
- Explain that a photon has momentum [including that the momentum is given by p=E/c (connect with the idea that light can exert a force)]
- describe that photoelectrons may be emitted from a metal surface when it is illuminated by electromagnetic radiation.
- describe and use the terms threshold frequency and threshold wavelength.
- explain photoelectric emission in terms of photon energy and work function energy.
- state and apply $hf = \phi + 1/2mv_{max}^2$
- explain why the maximum kinetic energy of photoelectrons is independent of intensity, whereas the photoelectric current is proportional to intensity.
- Juxtapose the evidence for light as a wave and as a particle [Explain that the photoelectric effect provides evidence for a particulate nature of electromagnetic radiation while phenomena such as interference and diffraction provide evidence for a wave nature].
- Analyze qualitatively the evidence provided by electron diffraction for the wave nature of particles.
- Explain and apply the de Broglie wavelength to solve problems [use $\lambda = h/p$ to solve problems].
- State that there are discrete electron energy levels in isolated atoms (e.g. atomic hydrogen).
- explain the appearance and formation of emission and absorption line spectra.
- use $hf = \Delta E$ to solve problems.
- Describe the Compton effect qualitatively.
- Explain the phenomena of pair production and pair annihilation.
- Explain how electron microscopes achieve very high resolution.
- State and explain Heisenberg's uncertainty principle qualitatively.
- Use the uncertainty principle to explain why empirical measurements must necessarily have uncertainty in them.